Noise Figure Degradation in Balanced Amplifiers

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***Abstract*— Balanced amplifiers suffer from the noise figure performance in comparison with a stand-alone amplifier even with an ideal input divider. The noise parameters degrade further with an imperfect divider. We present exact and approximate analytical results for the noise figure and noise parameters of the balanced amplifier in divider topology. Measurement results are also presented as a verification.**

***Index Terms*— Balanced amplifiers, mismatch, noise figure degradation, noise parameters.**

I. Introduction

LOW noise amplifiers (LNAs) are one of the most critical elements used in modern wireless communication systems, where a small noise figure is a common requirement. Obtaining a small noise figure and good input/output return loss values at the same time may not be possible using a single amplifier. Adding an isolator in front of the LNA [1] or using the balanced configuration [2], [3] solves the return loss problem. Ideally, balanced amplifiers conserve the gain of the single amplifier, while input/output return losses, linearity, and stability are improved [4]–[6] and a redundancy is provided.

Several papers investigated the effect of imperfections on the noise performance of a balanced amplifier. The noise figure was given [2], assuming that the amplifier gains are unbalanced. Kurokawa [3] explored the effect of the termination ports of the input/output couplers and Kerr [7] investigated the source impedance's effect on the noise figure of the balanced amplifier. In these cases, the input and output couplers were assumed to be ideal. An analytical formula was presented [8] to find the cascaded noise figure of the differential amplifier with baluns, assuming that the baluns have symmetrical losses. The analysis was improved [9] by adding the phase and amplitude imbalance in each arm. But, the return loss/isolation of the baluns and return loss of the amplifiers are all assumed to be zero.

While a balanced amplifier has many desirable properties, its topology may degrade the noise performance under certain conditions. In this paper, we investigate the noise figure of a balanced amplifier and find the conditions under which the noise figure is degraded compared with a stand-alone amplifier. We present exact and approximate results and verify the results with experiments.

II. Noise Figure of a Balanced Amplifier

The noise figure of a stand-alone amplifier, F, can be written in terms of its source impedance, Γ_s, as [10]

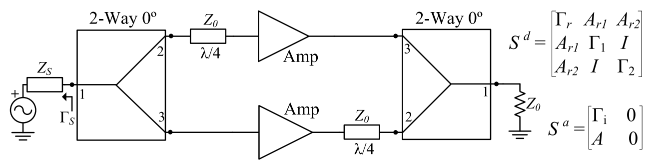

Fig. 1. Balanced amplifier built using two-way 0° power dividers.

$$F = F_m + 4r_n \frac{|\Gamma_s - \Gamma_o|^2}{(1-|\Gamma_s|^2)|1+\Gamma_o|^2} \tag{1}$$

where r_n is the normalized equivalent noise resistance, Γ_o is the optimal source reflection coefficient to obtain the minimum noise figure, F_m. Suppose that we use two such amplifiers with $\Gamma_o = 0$ and two identical two-way 0° power dividers/combiners to build a balanced amplifier as depicted in Fig. 1. S-parameters of the amplifiers (S^a) and the dividers (S^d) all specified with respect to Z_0 are also shown in matrix form. The parameters in S^d include the imperfections of the divider, such as finite ohmic loss, return loss, and isolation. If needed, any imperfection of the $\lambda/4$ line can also be represented by S^d. We assume that the gain of the amplifiers is sufficiently high and the output combiner is perfectly balanced so that the output combiner does not influence the noise figure.

To find an analytical expression for the noise figure of the balanced amplifier, we use the noise wave approach [10]. The noise waves and the correlation matrix for the divider can be found from the real part of its Y-parameters [11], while the noise waves for the amplifiers can be written in terms of their noise parameters [12]. The exact noise figure expression is determined using a symbolic computational package.[1] The resulting expressions are verified numerically by two separate linear microwave simulators.[2]

The noise figure of the balanced amplifier, F_d, with an ideal lossless input divider can be written as

$$F_d = F_m + 4\left(r_n + F_m \frac{|\Gamma_i|^2}{4}\right) \frac{|\Gamma_s|^2}{1-|\Gamma_s|^2}. \tag{2}$$

With a nonzero $|\Gamma_i|$, the input divider is terminated with different impedances at its output ports, and thus the isolation resistor of the divider contributes to the output noise. When $\Gamma_i = 0$, the symmetry is satisfied and the noise figure of the balanced amplifier is the same as that of the single amplifier. Note that the noise figure is independent of the phases of Γ_i and Γ_s. Fig. 2 has plots of noise figure variation as a function of $|\Gamma_s|$ for amplifiers with $F_m = 1$ dB. The noise figure of a balanced amplifier is degraded in comparison with

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[1] Symbolic toolbox of MATLAB, Mathworks, https://www.mathworks.com/

[2] AWR from AWR Corp. (http://www.awrcorp.com) and ADS from Keysight Technologies (www.keysight.com).

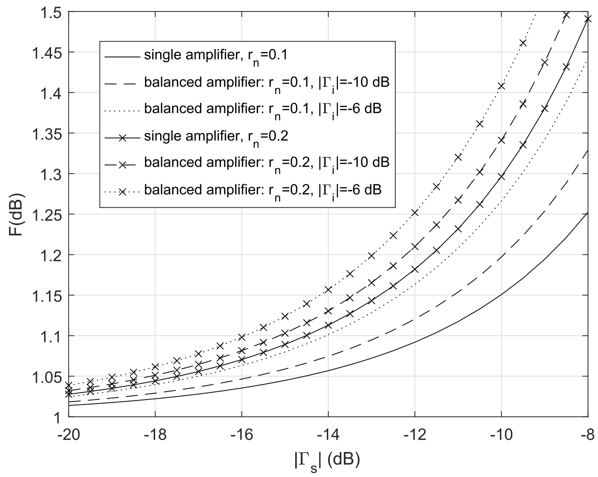

Fig. 2. Noise figures of single and balanced amplifiers with perfect dividers as a function of source reflection coefficient. Amplifiers have $F_m = 1$ dB, $\Gamma_o = 0$, and various r_n and $|\Gamma_i|$ values.

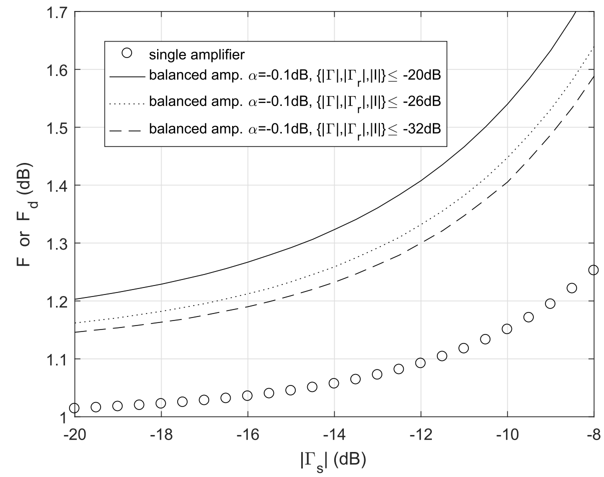

Fig. 3. Worst case noise figure as a function of source reflection coefficient for a balanced amplifier. The input divider has an ohmic loss of 0.1 dB, and return loss and isolation better than 20, 26, or 32 dB. The amplifiers have $|\Gamma_i| = -7$ dB, $F_m = 1$ dB, $r_n = 0.1$, and $\Gamma_o = 0$.

a single amplifier for the same source return loss. The normalized equivalent noise resistance is increased to $r_n + F_m|\Gamma_i|^2/4$, while F_m and $\Gamma_o = 0$ remain unchanged.

If the input divider is not perfect, there may be further degradation. We consider a perfectly symmetric but lossy divider. The full noise figure expression is too long to be given here. It can be written approximately as

$$F_d \approx \frac{F_m}{\alpha} + \frac{4\left(r_n + F_m\frac{|\Gamma_i|^2}{4}\right)|\Gamma_x|^2}{\alpha(1-|\Gamma_r|^2)(1-|\Gamma_s|^2)} + \frac{F_m(|\Gamma_s|^2 - 1 - |\Gamma_x|^2 + |1-\Gamma_r\Gamma_s|^2 + (1-|\Gamma_s|^2)|\Gamma_r|^2)}{\alpha(1-|\Gamma_r|^2)(1-|\Gamma_s|^2)}$$

with

$$\Gamma_x = \Gamma + I + 2A_r^2\Gamma_s \text{ and } \alpha \triangleq \frac{2|A_r|^2}{1-|\Gamma_r|^2}. \quad (3)$$

Here, α defines the ohmic loss of the divider and we have $\Gamma = \Gamma_1 = \Gamma_2$ and $A_r = A_{r1} = A_{r2}$. The approximation in (3) is accurate to within ± 0.03 dB, for $\{|\Gamma_r|, |\Gamma|, |I|\} \le -17$ dB, $|\Gamma_i| \le -7$ dB, $\alpha > -1$ dB and $|\Gamma_s| \le -10$ dB. Note that for $\Gamma_r = \Gamma = I = 0$ and $\alpha = 1$, (3) reduces to (2).

We ignore the last term of F_d expression in (3) and since $|A_r|^2(\Gamma + I) \approx -A_r^2\Gamma_r^*$, we let $|\Gamma_x| \approx |\Gamma_s - \Gamma_r^*|$, to find the approximate noise parameters of the balanced amplifier as

$$F_{\text{md}} \approx \frac{F_m}{\alpha}, \quad \Gamma_{\text{od}} \approx \Gamma_r^*$$
$$r_{\text{nd}} \approx \left(r_n + F_m\frac{|\Gamma_i|^2}{4}\right)\frac{|1+\Gamma_r|^2}{\alpha(1-|\Gamma_r|^2)} \quad (4)$$

where * is the conjugate operator. We observe that F_{md} increases, r_{nd} may increase or decrease, and Γ_{od} is no longer zero. Note that the noise parameters depend on α and Γ_r, but not on Γ or I. In (4), the accuracy of the parameters is given by: F_{md}: ± 0.005 dB, r_{nd}: $\pm 8\%$, $|\Gamma_{\text{od}}|$: ± 0.02, as long as $\{|\Gamma_r|, |\Gamma|, |I|\} \le -17$ dB, $\alpha > -0.2$ dB, and $r_n > 0.1$.

Fig. 3 presents the calculated noise figure of the balanced amplifier under different conditions. The curves are obtained using the exact noise figure expressions in a Monte Carlo simulation. Wilkinson dividers are built with two lossy transmission lines and an isolation impedance, the parameters of which have a statistical distribution. The 50 000 dividers with return loss and isolation better than 20, 26, or 32 dB and with an ohmic loss of 0.1 dB are considered. The phases of the source impedance and amplifier parameters are also chosen randomly for each simulation. The graphs show the worst case values of the noise figure. The noise figure of the stand-alone amplifier is also given for comparison.

Using the curves, we can investigate the possible benefit of a ferrite isolator in the noise figure when the source has a low return loss (e.g., an antenna). For example, suppose that the source has $|\Gamma_s| = -9.5$ dB. If an isolator with an insertion loss of 0.10 dB giving a return loss of 20 dB is inserted at the input of the single amplifier, the noise figure improves from 1.17 dB to $1.02 + 0.10 = 1.12$ dB. If we place the same isolator at the input of the balanced amplifier with 20-dB divider, we get $1.20 + 0.10 = 1.30$ dB instead of 1.58 dB.

We also investigated the effects of phase and amplitude imbalance of the input and output dividers. Note that any attenuation or phase error in the $\lambda/4$ transmission lines also generate these errors. We found that an amplitude or phase imbalance may result in a noise figure degradation or improvement depending on the phases of Γ_s and Γ_i. Assume that both dividers are lossless and unbalanced with amplitude error defined by $A_{r1}/A_{r2} = 1 + 2x$, where x is small. For an amplitude imbalance less than 0.4 dB, the noise figure lies in the range

$$F_d \approx F_m + 4\left(r_n + F_m\frac{|\Gamma_i|^2}{4} \pm F_m\left|\frac{\Gamma_i}{\Gamma_s}\right|x\right)\frac{|\Gamma_s|^2}{1-|\Gamma_s|^2}. \quad (5)$$

For example, with $F_m = 1$ dB, $r_n = 0.1$, $|\Gamma_i| = -7$ dB, and $|\Gamma_s| = -10$ dB, lossless dividers with 0.2 dB imbalance may result in at most 0.03-dB noise figure degradation or improvement.

If both dividers have the same phase error cancelling the total imbalance, the noise figure degradation is negligible. If only the input divider has a phase error defined by $A_{r1}/A_{r2} = e^{j\theta}$, the noise figure is in the range

$$F_d \approx F_m + 4\left(r_n + F_m\frac{|\Gamma_i|^2}{4} \pm F_m\left|\frac{\Gamma_i}{\Gamma_s}\right|\frac{\theta}{4}\right)\frac{|\Gamma_s|^2}{1-|\Gamma_s|^2}. \quad (6)$$

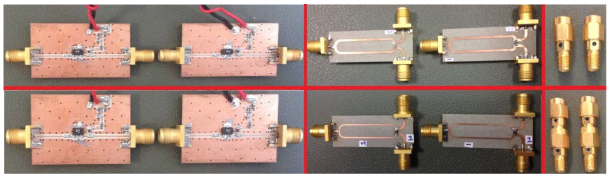

Fig. 4. Photograph of the amplifiers, divider pairs, and SMA line extenders used to build balanced amplifiers.

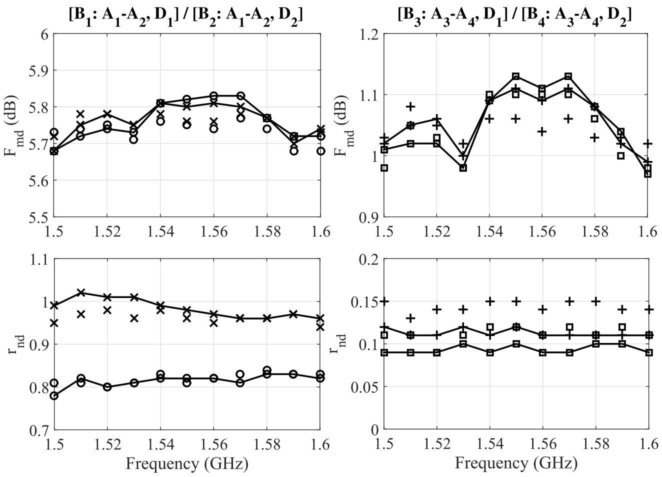

Fig. 5. Calculated (solid) and measured (points) F_{md} and r_{nd} for the balanced amplifiers B_1 (circle), B_2 (cross), B_3 (square), and B_4 (plus).

TABLE I

MEASURED PARAMETERS FOR AMPLIFIERS A_1/A_2 (GALI-84+) AND A_3/A_4 (PGA-103+) AT 1.55 GHz

Amp	$\|\Gamma_i\|$ (dB)	F_m (dB)	r_n
A_1/A_2	$-14.4/-13.9$	5.62/5.62	0.85/0.87
A_3/A_4	$-16.7/-17.4$	0.90/0.88	0.10/0.09

III. Experimental Results

To verify the equations of the noise parameters and the noise figure degradation in a balanced amplifier experimentally, two pairs of amplifiers, A_1–A_2 and A_3–A_4 are fabricated.[3] The inputs of the amplifiers are matched at 1.55 GHz to the optimal noise impedance ($\Gamma_o = 0$) to get the noise figure of F_m. Table I lists the parameters measured using Keysight PNA-X N5242A Network Analyzer at 1.55 GHz. Two dividers (with intentionally poor return loss and isolation) with an ohmic loss of 0.2 dB are manufactured. The dividers have $|\Gamma_r|$ values of -11 dB (D_1) and -25 dB (D_2), while $|\Gamma_1|$, $|\Gamma_2|$, and $|I|$ have nearly the same value of -17 dB. Four balanced amplifiers using different combinations of dividers and amplifier pairs (see Fig. 4) are tried: B_1 and B_3 using the divider D_1; B_2 and B_4 using the divider D_2. The noise parameters of the balanced amplifiers are measured using Keysight PNA-X N5242A in the frequency range 1.5–1.6 GHz. The same parameters are calculated using (4) from the measured individual amplifier parameters. The comparisons are presented in Figs. 5 and 6 indicating a good agreement. The measured values of Γ_{od} are equal to the measured values of Γ_r^*, confirming our theory.

[3] Mini-Circuits, NY 11235, USA, http://www.minicircuits.com

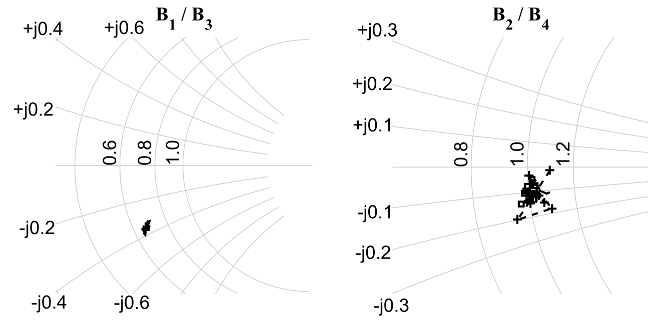

Fig. 6. Calculated (solid) and measured (points) Γ_{od} for the balanced amplifiers on the Smith chart in the frequency range 1.5–1.6 GHz.

IV. Conclusion

We analyze the noise figure for a balanced amplifier and we provide approximate analytical expressions for the noise figure and noise parameters. While a balanced amplifier provide an input port with a high return loss, it degrades the noise parameters even when an ideal divider is used when $|\Gamma_i|$ is nonzero. With an imperfect divider, there is further degradation in the noise parameters. This degradation is not only from the ohmic loss of the divider, but also from its input return loss. While a typical phase imbalance in the divider does not cause a problem, an amplitude imbalance may degrade the noise figure further. The presented graphs emphasize the need for a high-performance input divider to limit the noise figure degradation.

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